HEAT TRANSFER BETWEEN SOLID BODIES AND HIGH-TEMPERATURE GAS FLOWS

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A method based on the solution of a linearized third-order heat-conduction equation is proposed for the determination of intense thermal fluxes.

In studies of the flow of a plasma jet around solid bodies the question often arises of determining the magnitudes of both stationary and nonstationary heat fluxes incident on the surface being heated.

According to experimental data [1], it can be assumed that the heat flux in such situations varies according to a law:

$$q(\tau) = q_0(1 - e^{-\delta\tau}). \tag{1}$$

If we perform a numerical solution of the nonlinear heat-conduction equation with boundary condition (1), in which we assume q_0 = 3 kW/cm^2 and δ = 31.54 sec^{-1} [1], and

$$t(x, 0) = t_0, \tag{2}$$

where t_0 is a constant temperature, we can obtain the discrete temperature field as a function of coordinate and time. The results of such a calculation for a copper plate of length $R = 50 \cdot 10^{-3}$ m are shown in Table 1.

The problem of the present study then consists of proceeding from the numerically obtained temperature field to determine the original heat flux — Eq. (1). If their values coincide satisfactorily, the validity of the approach used will be confirmed.

The nonlinear heat-conduction equation

$$\rho_0 (C_0 + C_1\Theta) \frac{\partial\Theta}{\partial\tau} = \frac{\partial}{\partial x}\left[(\lambda_0 + \lambda_1\Theta)\frac{\partial\Theta}{\partial x}\right], \tag{3}$$

where

$$\varphi(\Theta) = \lambda_0 + \lambda_1\Theta, \tag{4}$$

$$\psi(\Theta) = C_0 + C_1\Theta \tag{5}$$

are functions expressing the linear temperature dependence of the coefficients of thermal conductivity and specific heat, respectively, while $\Theta(x, \tau) = t(x, \tau) - t_0$ can be written in the form

$$\frac{\partial}{\partial\tau}\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right) - a_0\frac{\partial^2}{\partial x^2}\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right) = -\frac{1}{2}\left(\frac{C_1}{C_0} - \frac{\lambda_1}{\lambda_0}\right)\frac{\partial^2\Theta}{\partial\tau^2} = \Phi(x, \tau). \tag{6}$$

Now, assuming the function $\Phi(x, \tau)$ continuous and n-times differentiable with respect to the spatial coordinate x, Eq. (6) can be written as

$$\frac{\partial}{\partial\tau}\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right) = a_0\frac{\partial^2}{\partial x^2}\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right) + \sum_{n=0}^{\infty}\frac{1}{n!}\Phi^{(n)}(0, \tau)(x - x_0)^n, \tag{7}$$

where $\sum_{n=0}^{\infty}\frac{1}{n!}\Phi^{(n)}(0, \tau)(x - x_0)^n$ is an expansion of the function $\Phi(x, \tau)$ in the spatial coordinate x at the arbitrary point $x = x_0$.

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TABLE 1. Plate Temperature Field vs Time and Coordinate, °C

τ, sec	x, mm					τ, sec	x, mm				
	1	2	3	5	15		1	2	3	5	15
0,02	17,6	6,2	1,9	0,1	0	0,18	278,8	216,8	165,9	92,5	1,6
0,04	54,3	27,8	13,3	2,4	0	0,20	302,0	238,5	185,8	107,9	2,6
0,06	94,0	56,2	32,0	8,9	0	0,22	323,7	259,1	204,8	123,1	4,0
0,08	131,9	86,1	54,1	19,1	0	0,24	344,4	278,7	223,1	138,0	5,6
0,10	166,8	115,4	77,6	32,0	0	0,26	364,0	297,5	240,7	152,6	7,5
0,12	198,5	143,2	100,9	46,3	0,2	0,28	382,8	315,5	257,7	166,9	9,7
0,14	227,5	169,3	123,4	61,5	0,4	0,30	400,8	332,8	274,1	180,9	12,2
0,16	254,1	193,7	145,1	77,0	0,9						

Analogously, as was done in [2], limiting ourselves to n terms on the right side of Eq. (7), after n-fold differentiation we obtain a linear differential equation of order n + 2:

$$\frac{\partial^{n+1}}{\partial x^n \partial \tau}\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right) = a_0 \frac{\partial^{n+2}}{\partial x^{n+2}}\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right). \tag{8}$$

In [2], third- and fourth-order linear heat-conduction equations were also compared with the exact solution of the nonlinear heat-conduction equation and it was shown that the former provide completely satisfactory accuracy for practical use.

Thus, limiting ourselves to a third-order linear heat-conduction equation, the mathematical formulation of the problem under consideration will be as follows:

$$\frac{\partial^2}{\partial x \partial \tau}\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right) = a_0 \frac{\partial^3}{\partial x^3}\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right), \tag{9}$$

$$\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right)\Bigg|_{\tau=0} = 0, \tag{10}$$

$$\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right)\Bigg|_{x=R_1} = \varphi_1(\tau) + \frac{\lambda_1}{2\lambda_0}\varphi_1^2(\tau) = \psi_1(\tau), \tag{11}$$

$$\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right)\Bigg|_{x=R_2} = \varphi_2(\tau) + \frac{\lambda_1}{2\lambda_0}\varphi_2^2(\tau) = \psi_2(\tau), \tag{12}$$

$$\left(\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2\right)\Bigg|_{x=R_3} = \varphi_3(\tau) + \frac{\lambda_1}{2\lambda_0}\varphi_3^2(\tau) = \psi_3(\tau), \tag{13}$$

where $\varphi_1(\tau)$, $\varphi_2(\tau)$, and $\varphi_3(\tau)$ are temperatures taken from the numerical solution at points R_1, R_2, and R_3, respectively.

Taking the Laplace transform, we obtain a solution for the image in the form

$$T = \psi_1(s) - (\psi_1(s) - \psi_2(s)) \frac{\mathrm{sh}\sqrt{\frac{s}{a_0}}\cdot\frac{x-R_1}{2}\,\mathrm{sh}\sqrt{\frac{s}{a_0}}\cdot\frac{R_3-x}{2}}{\mathrm{sh}\sqrt{\frac{s}{a_0}}\cdot\frac{R_2-R_1}{2}\,\mathrm{sh}\sqrt{\frac{s}{a_0}}\cdot\frac{R_3-R_2}{2}}$$

$$-(\psi_1(s) - \psi_3(s)) \frac{\mathrm{sh}\sqrt{\frac{s}{a_0}}\cdot\frac{x-R_1}{2}\,\mathrm{sh}\sqrt{\frac{s}{a_0}}\cdot\frac{x-R_2}{2}}{\mathrm{sh}\sqrt{\frac{s}{a_0}}\cdot\frac{R_3-R_1}{2}\,\mathrm{sh}\sqrt{\frac{s}{a_0}}\cdot\frac{R_3-R_2}{2}} \tag{14}$$

Then transforming to the original and limiting ourselves to only third-order derivatives of the functions $\varphi_1(\tau)$, $\varphi_2(\tau)$, and $\varphi_3(\tau)$, the solution of system (9)-(13) can be represented as

$$\Theta + \frac{\lambda_1}{2\lambda_0}\Theta^2 = \psi_1(\tau) - (\psi_1(\tau) - \psi_2(\tau))\frac{(x-R_1)(R_3-x)}{(R_2-R_1)(R_3-R_2)} - (\psi_1(\tau) - \psi_3(\tau))\frac{(x-R_1)(x-R_2)}{(R_3-R_1)(R_3-R_2)} -$$
$$-(\psi_1'(\tau) - \psi_2'(\tau))\frac{(x-R_1)(R_3-x)(R_2-x)(R_3-R_2+R_1-x)}{12a_0(R_2-R_1)(R_3-R_1)} -$$
$$-(\psi_1'(\tau) - \psi_3'(\tau))\frac{(x-R_1)(x-R_2)(R_3-x)(R_2+R_1-R_3-x)}{12a_0(R_3-R_1)(R_3-R_2)}. \quad (15)$$

Differentiating Eq. (15) on the left and right with respect to x and setting the result equal to zero, we obtain

$$q(\tau) = \lambda_0\left[(\psi_1(\tau) - \psi_2(\tau))\frac{R_3+R_1}{(R_3-R_2)(R_2-R_1)} - \right.$$
$$-(\psi_1(\tau) - \psi_3(\tau))\frac{R_2+R_1}{(R_3-R_2)(R_3-R_1)} +$$
$$+(\psi_1'(\tau) - \psi_2'(\tau))\frac{(R_3+R_1)(R_3R_2+R_3R_1+R_2R_1-R_2^2)}{12a_0(R_3-R_2)(R_2-R_1)} -$$
$$\left.-(\psi_1'(\tau) - \psi_3'(\tau))\frac{(R_2+R_1)(R_3R_2+R_3R_1+R_2R_1-R_3^2)}{12a_0(R_3-R_2)(R_3-R_1)}\right] \quad (16)$$

for determination of the heat fluxes incident on the surface of the solid bodies.

From Eq. (16) it follows that the heat-flux function $q(\tau)$ depends on the functions $\varphi_1(\tau)$, $\varphi_2(\tau)$, and $\varphi_3(\tau)$, which are usually taken from experiment, and on the first derivatives of these functions. However, numerical differentiation, in contrast to numerical integration, is within the class of incorrectly posed problems. Therefore, to obtain reliable results in these cases it is necessary to use approaches developed in the theory of numerical differentiation [3-7]. In particular, in [7], methods of smoothing the desired function and finding its derivatives up to second order, based on the method of least squares and distinguished by its simplicity and satisfactory accuracy, are described.

Figure 1 presents results of determining the heat flux — Eq. (1) — obtained with Eq. (16) and the temperature field presented in Table 1. The first derivatives of the functions $\varphi_1(\tau)$, $\varphi_2(\tau)$, and $\varphi_3(\tau)$ were calculated by the method described in [7].

It follows from Fig. 1 that the calculated data agree better with the original equation (1) the closer to the heating surface the values of temperature and its derivatives are taken — curve 2, where the divergence of results is not greater than 2%. Curves 3, 4, and 5 diverge from curve 1 by about 5%, which may be due to degradation of the degree of temperature-field approximation at the point x = 0.

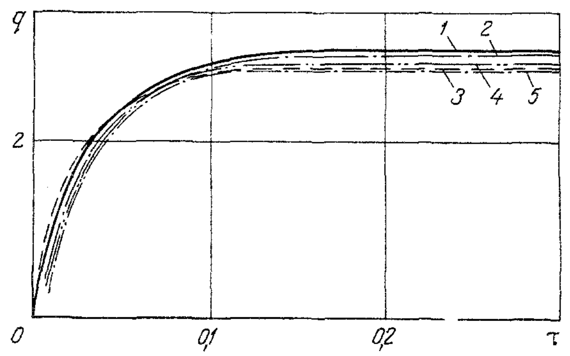

Fig. 1. Determination of heat flux: curve 1) original equation (1); 2, 3, 4, 5) using Eq. (16) with $R_{1,2,3}$ = 1, 3, and 5 mm; $R_{1,2,3}$ = 1, 5, and 15 mm; $R_{1,2,3}$ = 2, 5, and 15 mm; and $R_{1,2,3}$ = 3, 5, and 15 mm, respectively. q, kW/cm^2; τ, sec.

It should be noted that all cases of determining heat flux considered here give good results, while the proposed method of flux determination is distinguished by simplicity and can be recommended for practical use.

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TWO-PHASE ZONE DURING CRYSTALLIZATION OF A BINARY MELT

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A method is developed for calculating the solid–liquid zone, which is intermediate between the regions of the liquid and the solid phases, of a solidifying binary melt. Melts of Fe–C with different initial carbon contents were chosen as the binary melt.

The thermal and diffusional processes occurring in the two-phase zone have considerable importance in the formation of the micro- and macrostructures of ingots and castings.

The study of the kinetics of the movement of the zone under different conditions of crystallization and the question of the extent of the two-phase zone have great importance, since they determine the main technological properties of the metals.

The processes of a two-phase zone were studied in a region for which a diagram (of half) is presented in Fig. 1a.

The two-phase state of the medium at each point η is conveniently characterized by the function $l(\eta, \mathrm{Fo})$, representing the fraction of the solid phase in the liquid melt at the isotherm with the coordinate η at the time Fo [1, 2]. All the isotherms are assumed to be parallel planes perpendicular to the $O\eta$ axis. The coefficients of thermal conductivity λ and heat capacity c_p are the same and are equal for the solid and liquid phases. The concentration $C(\eta, \mathrm{Fo})$ of the admixture at one isotherm is the same at all points of the melt [3]. The character of the occurrence of diffusional processes allows one to assume that the rate of diffusion of the admixture into the solid phase is small in comparison with the rate of diffusion of the same admixture into the liquid melt. Crystals develop in the liquid phase in the process of crystallization. As this happens, the latent heat of fusion is released, depending on the rate of change of the amount of solid melt. It is expedient to treat the effect of the developing crystals on the fields of temperature $T(\eta, \mathrm{Fo})$ and concentration $C(\eta, \mathrm{Fo})$ as the action of additional sources of heat and admixture. Moreover, we assume that concentration supercooling is absent within the two-phase zone. Mathematically, this means that the concentration and temperature are connected by the equation for the liquidus line on the equilibrium diagram of state [1, 2].

With allowance for the foregoing and for simple transformations of the equations of the quasiequilibrium theory of a two-phase zone [1, 2], the processes of mass and heat transfer

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